

SOLUTION OF THE PROBLEM OF THE MONTH

I suppose that Eratostene known the earth diameter D_{earth} (by the shadow of the spear...sun elevation...the experiment of Alessandria...);

The brilliance is an invariant and

$$P = B A \Omega$$

Where: P =power; B =brilliance; A =area of the body; Ω =solid angle.

I indicate α_m the albedo of the body like the ratio between the reflected power and incident power. And also:

$B_{m/e}$ =brilliance of the moon illuminated by the earth;

$B_{m/s}$ =brilliance of the moon illuminated by the sun;

sun and earth are lambertian source;

$$B_{m/e} = \alpha_m B_{earth} \Omega_{earth} / \pi ;$$

$$B_{m/s} = \alpha_m B_{sun} \Omega_{sun} / \pi ;$$

the ratio is equal to:

$$B_{m/e} / B_{m/s} = B_{earth} \Omega_{earth} / B_{sun} \Omega_{sun} ;$$

but the brilliance of the earth is due the sun (by the albedo α_e). In fact:

$$B_{earth} = \alpha_e B_{sun} \Omega_{sun} / \pi ;$$

if the quantity α_e is known then:

$$B_{m/e} / B_{m/s} = \alpha_e \Omega_{earth} / \pi ;$$

but geometrically

$$\Omega_{earth} = \pi \sin^2 \theta$$

$$\text{and } \theta = D_{earth} / L_{moon_earth}$$

and $\theta \ll 1$ (and using the Taylor series)

$$\Omega_{earth} = \pi (D_{earth} / L_{moon_earth})^2$$

at finally:

$$L_{moon_earth} = [\alpha_e D_{earth}^2 B_{m/s} / B_{m/e}]^{1/2}$$

